**2.** Suppose that A is the set of sophomores at your school and B is the set of students in discrete mathematics at your school. Express each of these sets in terms of A and B.

**a)** the set of sophomores taking discrete mathematics in your school

**A ∩ B**

**b)** the set of sophomores at your school who are not taking discrete mathematics

**A - B**

**c)** the set of students at your school who either are sophomores or are taking discrete mathematics

**A U B**

**d)** the set of students at your school who either are not sophomores or are not taking discrete mathematics

**4.** Let A = {a, b, c, d, e} and B = {a, b, c, d, e, f, g, h}.

Find

**a)** A ∪ B.

**{a, b, c, d, e, f, g, h}**

**b)** A ∩ B.

**{a, b, c, d, e}**

**c)** A − B.

**none φ**

**d)** B − A.

**{f, g, h}**

**12.** Prove the first absorption law from Table 1 by showing that if A and B are sets, then A ∪ (A ∩ B) = A.

**Show each side is a subset of the other.**

**-In a union, x ∈ A or x ∈ A∩B.**

**-Intersection states that either x ∈ A or x ∈ A and x ∈ B, so x ∈ A.**

**-We can then say that A∪(A∩B)⊆A**

**-if x ∈ A then x by the definition of union, ∈A∪(A∩B), because x ∈ A can also be x ∈ A and x ∈ B, which also means that A⊆A∪(A∩B)**

**- We can therefore conclude that A ∪ (A ∩ B) = A**

**16.** Let A and B be sets. Show that

**a)** (A ∩ B) ⊆ A.

Show x∈(A∩B)

**- x∈A and x∈B by intersection**

**-because x∈A, we can then conclude (A∩B)⊆A, proving the statement**

**b)** A ⊆ (A ∪ B)

Show x ∈ A

**-x ∈ A or x ∈**

**-by union we know that x ∈ A∪B,**

**- because x ∈ A∪B we can thus conclude that A ⊆ (A ∪ B)**

**c)** A − B ⊆ A.

**- x ∈ A - B**

**-x ∈ A and x ∉B because of their difference, so x ∈ A**

**because x ∈ A, we can conclude A − B ⊆ A.**

**d)** A ∩ (B − A) = ∅.

Show contradiction

**- A ∩ (B − A) does not = ∅**

**-x ∈ A and x ∈ (B−A) by union**

**- through difference, we can find that x ∈ A, x ∈ B, but also x ∉ A.**

**-This contradiction proves that A ∩ (B − A) = ∅.**

**e)**  A U(B − A) = AU B.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| A | B | B - A | A U (B - A) | A U B |
| 1 | 1 | 0 | **1** | **1** |
| 1 | 0 | 0 | **1** | **1** |
| 0 | 1 | 1 | **1** | **1** |
| 0 | 0 | 0 | **0** | **0** |

**Because A U (B - A) and A U B have the same values, we can conclude that they are equal.**

**18.** Let A, B, and C be sets. Show that

**a)** (A ∪ B) ⊆ (A ∪ B ∪ C).

**-if x ∈ (A∪B), then by union x ∈ A or x ∈ B.**

**-x then is also ∈ C**

**-Again using union, we can conclude then that because x ∈ (A u B u C) that**

**(A ∪ B) ⊆ (A ∪ B ∪ C).**

**b)** (A ∩ B ∩ C) ⊆ (A ∩ B).

**- by interstection, x ∈ A, x ∈ B, x ∈ c, which would also give that x ∈ (A ∩ B)**

**-We can then conclude that (A ∩ B ∩ C) ⊆ (A ∩ B).**

**c)** (A − B) − C ⊆ A − C.

**- To show the left, we can assume x ∈ (A−B)−C.**

**-Through difference, this returns x ∈ A, while x ∈ B and x ∉ C.**

**-Intersection then gives us x ∈ (A ∩ B).**

**-We can thus conclude that (A ∩ B ∩ C) ⊆ (A ∩ B).**

**d)** (A − C) ∩ (C − B) = ∅.

Show the contradiction to prove. (That there is a value that exists so that (A − C) ∩ (C − B) ≠ ∅.)

**-By intersection, we can show x ∈ (A - C) and x ∈ (C - B).**

**-By difference, x ∈ A and x ∉ C**

**-By difference, x ∈ C and x ∉ B**

**-Because x ∉ C and x ∈ C contradict eachother, we can conclude that (A − C) ∩ (C − B) = ∅.**

**e)** (B − A) ∪ (C − A) = (B ∪ C) − A.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| A | B | C | B - A | C - A | B U C | (B - A) U (C - A) | (B U C) - A |
| 1 | 1 | 1 | 0 | 0 | 1 | **0** | **0** |
| 1 | 1 | 0 | 0 | 0 | 1 | **0** | **0** |
| 1 | 0 | 1 | 0 | 0 | 1 | **0** | **0** |
| 1 | 0 | 0 | 0 | 0 | 0 | **0** | **0** |
| 0 | 1 | 1 | 1 | 1 | 1 | **1** | **1** |
| 0 | 1 | 0 | 0 | 1 | 1 | **1** | **1** |
| 0 | 0 | 1 | 1 | 0 | 1 | **1** | **1** |
| 0 | 0 | 0 | 0 | 0 | 0 | **0** | **0** |

**Because the values for (B - A) U (C - A) are equal to (B U C) - A, we can conclude that (B − A) ∪ (C − A) = (B ∪ C) − A.**

**20.** Show that if A and B are sets with A ⊆ B, then

**a)** A ∪ B = B.

**-Assume that A ⊆ B**

**-if x ∈ A ∪ B then x ∈ A or x ∈ B**

**-Assume x ∈ A**

**-if x ∈ A, then because A ⊆ B, x must also be ∈ B**

**- This shows B ⊆ A U B, and thus that B = A U B.**

**-Together, this proves B = A U B**

**b)** A ∩ B = A.

**-Assume that A ⊆ B**

**- If x ∈ A ∩ B, then x ∈ A and x ∈ B by intersection, so x ∈ A.**

**-This shows A ∩ B ⊆ A.**

**-if x ∈ A, then x ∈ B, too, so x ∈ A ∩ B.**

**-This shows A ⊆ A ∩ B, and thus that A = A ∩ B.**

**- If x ∈ A, then x ∈ A ∩ B, so x ∈ A and x ∈ B.**

**- This proves A ⊆ B, so we can conclude that if A ⊆ B, where A and B are sets, then A ∩ B = A.**